A FEATURE OF SHOCK COMPRESSIBILITY WITH DISAPPEARANCE OF THE TWO-WAVE CONFIGURATION

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It is a familiar fact that in certain cases of shock compression of matter characterized by thermodynamic equilibrium, a two-wave configuration is formed instead of a single shock wave; a second shock wave moves in the wake of the first wave, which is of constant amplitude, but does not overtake it. Such a two-wave configuration may be observed, in particular, in plastic flow and in phase transitions [1, 2].

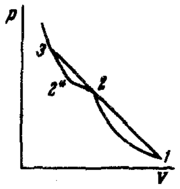

Fig. 1

The pressure P is usually represented as a function of the specific volume V of the shock-compressed substance in its initial state 1 as shown in Fig. 1, with one discontinuity in the case of plasticity (point 2) and with two in the case of a phase transition of the first kind (points 2 and 2*, the discontinuity at point 2* being associated with the completion of the phase transition; the two-wave configuration corresponds to any point on the part of the curve 2-3). Now, in fact, at point 3, where the two-wave configuration vanishes, there is also a discontinuity on the P(V) curve.* The reason for this is that the function P(V) is described by the shock adiabatic curve with initial state 2 for $P_2 < P \leq P_3$, and for $P > P_3$ by the shock adiabatic curve with initial state 1. It may easily be seen that these are two different adiabatic curves intersecting at point 3 (we shall call them adiabatic curves II and I, respectively). Actually, at point 3 where the adiabatic curve II intersects the continuation of chord 1-2, and where the first and second waves have equal velocities, the two-wave configuration may be considered as a single stationary discontinuity satisfying the conservation laws. This also means that point 3 belonging to adiabatic curve II also belongs to adiabatic curve I.

We shall find the direction of the discontinuity in the P(V) curve at the point 3. By the definition of point 3, a two-wave configuration is realized everywhere in the interval $P_2 < P < P_3$, and so in the interval P considered there are no other points where the adiabatic curve II intersects the continuation of the chord 1-2, except the point 3, and consequently there are no other points of intersection of the adiabatic curves I and II. Thus in order to investigate the direction of the discontinuity at point 3 it suffices to determine the relative position of the adiabatic curves I and II at the point 2.

We shall do this by calculating the differential enthalpies on the shock adiabatic curves I and II for a pressure $P_2 + dP > P_2$. Denoting the enthalpies and the volumes of the final state on shock adiabatic curves I and II by H_I, V_I, H_{II}, V_{II}, respectively, and differentiating Hugoniot's equation at point 2, we find

$$dH_I = {}^1/_2 (V_2 + V_1)\, dP + {}^1/_2 (P_2 - P_1)\, dV_I, \qquad dH_{II} = V_2 dP\,.$$

Subtracting the first equation from the second, we obtain

$$d(H_{II} - H_I) = {}^1/_2 (V_2 - V_1)\, dP - {}^1/_2 (P_2 - P_1)\, dV_I\,. \qquad (1)$$

Comparing (1) with the condition for the formation of the two-wave configuration for $P = P_2 + dP$, which is of the form

$$\frac{dP}{dV_I} > \frac{P_2 - P_1}{V_2 - V_1}, \qquad (2)$$

we arrive at the inequality $d(H_{II} - H_I) > 0$, or, expressing dH in terms of the differential entropy and pressure,

$$T_2 dS_{II} + V_2 dP - T_2 dS_I - V_2 dP = T_2 (S_{II} - S_I)_{P_2+dP} = T_2 \left(\frac{\partial S}{\partial V}\right)_P (V_{II} - V_I) = T_2 \left(\frac{\partial P}{\partial T}\right)_V (V_{II} - V_I) > 0\,. \qquad (3)$$

Here $V_{II} - V_I$ is the difference in volumes on the shock adiabatic curves at a pressure $P_2 + dP$.

According to (3), the relative position of the adiabatic curves I and II is determined by the sign of $(\partial P/\partial T)_V$ in the region where the two-wave configuration occurs in the neighborhood of point 2. (The derivative $(\partial P/\partial T)_V$ may not have a unique sign at the point 2 itself, for example, as the result of a phase transition; if the two-wave configuration is associated with a phase transition, then for $dP > 0$ both shock adiabatic curves fall in the two-phase region.)

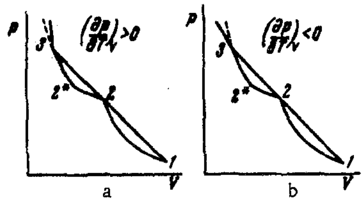

Fig. 2

The position of the shock adiabatic curves for positive and negative $(\partial P/\partial T)_V$ and the corresponding directions of the discontinuity at the point where the two-wave configuration disappears (point 3) are given in Fig. 2. The continuation of the adiabatic curve II is shown by a broken line. We note that in the case where a two-wave configuration does not arise on the shock adiabatic curve at point 2, then, as is well known [4], the relative position of the shock adiabatic curves (the shock adiabatic and isentropic curves) is also determined by the sign of $(\partial P/\partial T)_V$, but turns out to be the direct opposite.

REFERENCES

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*This fact was noted, for example, in the review [3] which has recently appeared.